# A New Model for Converting Short Span Compression with Other Measurements to Ring Crush

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## ABSTRACT

Linerboard and medium for corrugated packaging has been traditionally marketed on the basis of Ring Crush (RCT) specifications. The current industry trend has replaced manual testing by automated in-line testing which excludes RCT but has basis weight, an ultrasonic stiffness index (TSI), and short span compression (SCT) commonly available. This paper shows how using a combination of these measurements with a non-linear mechanistic theory, equivalent RCT values can be calculated with a good reliability. The model is applied using a wide range of medium and linerboards obtained from 2 mills.

## INTRODUCTION

Linerboard and medium are traditionally marketed by ring crush compression (RCT) as the primary strength characteristic. The practice continues despite that it is established that the optimization of strength properties through furnish changes, wet pressing, refining, etc., is best addressed using short span compression (SCT) [1]. The requirement of paperboard converters is to qualify incoming linerboard and medium based on historic RCT values. Automated testing machines are currently replacing many testing labs in production facilities but do not have RCT measurements available. Therefore, there are occasions where SCT is measured but RCT is not. A conversion from SCT to RCT using available measurements is proposed in this paper.

## BASIS OF THE MODEL

A model is proposed based on intuitive reasoning. Similar to the development of the McKee equation for the vertical compression strength of boxes (BCT) [2], RCT, the maximum load measured in the axial crushing of a thin cylindrical shell, is proposed to be a combination of compression and buckling failure. Observation of crushed RCT specimens as in Fig.1, indicate a characteristic buckling pattern along with creased failure.

Ring Crush specimen after testing; buckling and
rolling edges clearly visible

**Figure 1. A typical RCT test specimen after crushing. A characteristic buckling pattern and a wrinkled top edge is present. (Mike Schaepe photo).**

In contrast, the geometry of the SCT method tests a 0.7 mm length of a specimen which excludes any bending. The failure in this case consists only of a crease formed by the delamination of fiber layers through the thickness of the sheet [3] in the testing free span area. Therefore, SCT is considered as the correct measure of the compression strength .

The buckling load $\sigma_{cr}$ of a thin shell ring based on the analyses of Timoshenko and Gere [4] is:

$$\sigma_{cr} = \frac{E_{CD}}{\sqrt{1-\nu_{12}\nu_{21}}}\frac{t}{R} \qquad (1)$$

with $E_{CD}$ the CD modulus, $t$ the thickness of the test specimen, $R$ is the radius of the ring fixed at 24.2 mm, the Poisson ratio term under the square root sign adds a few percent correction to the buckling load and does not vary significantly with different papers [5]. The criterion for this equation to be valid is that the ratio of the buckling column height $l$ over $R$ must be sufficient to fit at least one buckling wave and is stated as:

$$l = 1.72\sqrt{Rt} \qquad (2)$$

This criterion is fulfilled for the standard RCT strip width of 12.4 mm of which 6 mm protrudes outside of the fixture base. The general form of the proposed combined compression/buckling model is similar to the BCT McKee equation:

$$RCT = C(SCT)^b(\sigma_{cr}R)^{1-b} \qquad (3)$$

with $C$ and $b$ being empirical constants. To determine the buckling load, it is convenient to realize that $E_{CD}\, t$ is the tensile stiffness. The speed of sound in the CD $V_{CD}$ is related to the modulus through the relation

$$E_{CD} = \rho \mathrm{V}_{\mathrm{CD}}^2(1-\nu_{12}\nu_{21}) \qquad (4)$$

where $\rho$ is the density, basis weight divided by the caliper $\beta/t$. Therefore the model for RCT can rewritten in terms of the basis weight, and speed of sound as:

$$RCT = C(SCT)^b(\beta V_{CD}{}^2)^{1-b} \qquad (5)$$

Measurement of $V_{CD}{}^2$ are available using commercially supplied instruments such as the TSO (Tensile Stiffness Orientation, AB Lorentzen and Wettre, Kista, Sweden) and others. These instruments have been customarily utilized to generate stiffness orientation roll CD strip profiles of tensile stiffness orientation for optimizing paper machine headbox flows. The TSO also readily provides numerical values of *TSI_CD* and *TSI_M,* which are proportional to $V^2$ in the two principal directions. Repeatability of ultrasonic $V_{CD}{}^2$ measured to be 3% or less for linerboard, for mechanically measured stiffness it is about 5 % typically.

## USING ULTRASONIC MEASUREMENTS

Ultrasonic measurements offer a quick convenient replacement for many mechanical measurements [6] and are suggested here for the current purpose. Utilizing TSO or other sonically based stiffness data for other than orientation is an additional opportunity not realized in many installations. The correspondence between ultrasonic and mechanical measurements requires to be established since ultrasonic results are equipment specific. The IPST in-plane robot based ultrasonic measurement developed in the 1980's is based on a paired difference method which despite being comparatively time consuming, excludes artifacts that are introduced through electronic delays, sample-transducer coupling, signal processing and other details.

Accordingly, measurements of the specific stiffness $V^2$ from the legacy IPST instrument were compared with *TSI_CD* and *TSI_MD* values produced from a TSO instrument using a variety of commercial paper and plastic sheet samples with a wide range of basis weights. Figure 2 shows that *TSI_CD* and *TSI_MD* values correlate with IPST $V^2$ (km/s)$^2$ and are about 5% higher. The correlation requires the TSO instrument to be calibrated using its supplied Mylar laminated sheet.

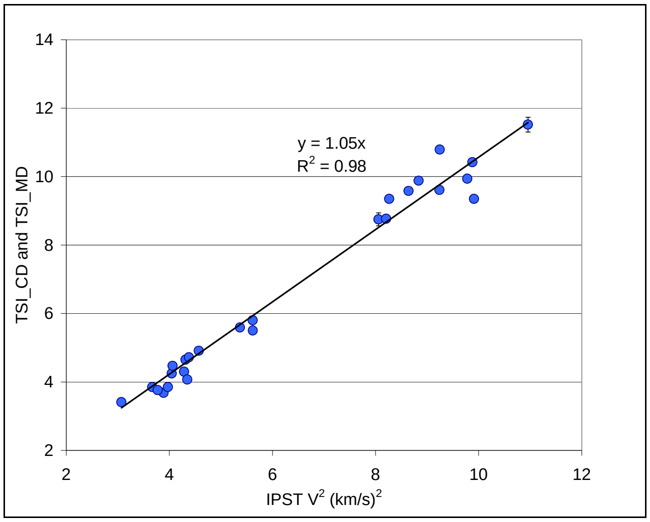

**Figure 2. Comparison of TSI output values and corresponding measurements of the IPST ultrasonic in-plane**

Comparison of *TS_CD x β* was made with the tensile stiffness $E_{CD}t$ measured on the same selected paper set using T 494 tensile tests on an Instron testing machine model 1122 using Series IX software. Results shown in Figure 3 indicate that *TSI_CD x β* is lager than the mechanically measured equivalent tensile stiffness by 75 % which is an expected result consistent with previous comparisons of ultrasonic to mechanically measure physical properties [7]. The correlations of the TSO outputs with other measurements shown in Figures 2 and 3 provide the confidence to use TSO data in the *RCT* model equation (5).

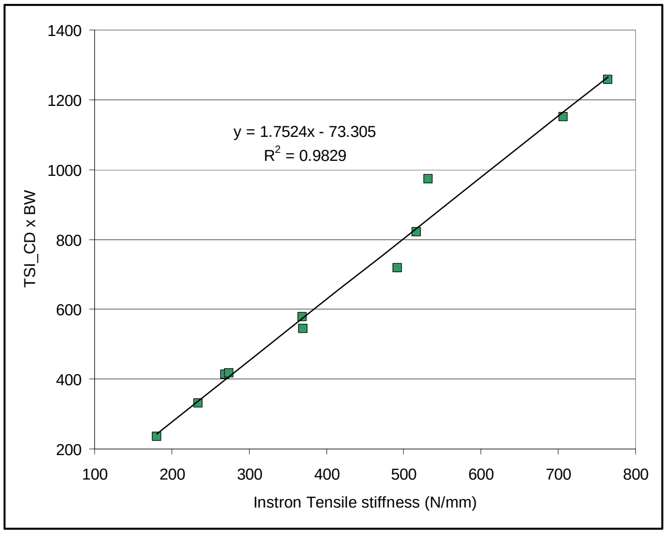

**Figure 3. Comparison of the TSI_CD x BW with tensile stiffness measured by mechanical testing.**

## COMPARISON OF RCT WITH SCT

Samples of unbleached softwood kraft linerboard representing a wide range of basis weights were obtained from two mills in southeastern US. The sample set was also augmented by a series of OCC medium of a range of basis weights. Measurements of relevant physical properties are reported in Table 1. Comparison of *SCT* and *RCT* with basis weight is shown in Figure 4. *SCT* tracks basis weight as expected however RCT appears to deviate from linearity with increasing basis weight likely due to a larger preponderance of edge rolling at larger basis weights. Figure 4 also indicates that *RCT* is less than *SCT* by almost a factor of two when using the same units of lb/in, although both measurements are supposed to be measuring the same property. At low basis weights corresponding to lower calipers, more bending can be expected. At higher basis weights, increased edge wrinkling compounds the deviation of RCT from linearity with basis weight in Figure 4.

**Table 1. Data for linerboard and medium samples. Liners A are from one mill, liners B from another.**

| Sample | Caliper<br>(mm) | Basis Weight<br>($g/m^2$) | TSI-CD<br>$(km/s)^2$ | CD SCT<br>(lb/in) | Ring Crush<br>(lb) |
|---|---|---|---|---|---|
| Liner A1 | 0.232 | 150.1 | 4.59 | 18.6 | 61.5 |
| Liner A2 | 0.268 | 171.7 | 4.88 | 22.4 | 76.1 |
| Liner A3 | 0.305 | 195.3 | 5.46 | 22.1 | 101.2 |
| Liner A4 | 0.392 | 249.5 | 4.7 | 30.8 | 118.4 |
| Liner A5 | 0.420 | 275.7 | 4.9 | 31.2 | 122.4 |
| Liner A6 | 0.476 | 293.9 | 4.69 | 33.5 | 129.6 |
| Liner B1 | 0.600 | 370.4 | 4.43 | 46.1 | 158.4 |
| Liner B2 | 0.427 | 277.1 | 5.13 | 36.3 | 126.5 |
| Liner B3 | 0.725 | 437.1 | 3.78 | 51.2 | 161.5 |
| Liner B4 | 0.327 | 206.3 | 5.51 | 26.2 | 98.5 |
| Liner B5 | 0.663 | 393.1 | 3.96 | 40.8 | 160.2 |
| Liner B6 | 0.260 | 167.0 | 5.24 | 21.7 | 74.3 |
| Liner B7 | 0.524 | 323.8 | 3.97 | 34.4 | 125.4 |
| Liner B8 | 0.229 | 149.6 | 4.55 | 16.1 | 62.0 |
| Liner B9 | 0.322 | 195.8 | 5.43 | 28.6 | 98.8 |
| Liner B10 | 0.319 | 205.4 | 4.68 | 22.2 | 87.8 |
| Liner B11 | 0.203 | 128.9 | 5.13 | 14.3 | 39.0 |
| Liner B12 | 0.259 | 177.6 | 4.97 | 17.2 | 65.7 |
| medium 1 | 0.146 | 78.5 | 3.58 | 7.5 | 15.2 |
| medium 2 | 0.192 | 88.3 | 3.76 | 8.6 | 21.9 |
| medium 3 | 0.197 | 95.7 | 4.12 | 10.0 | 25.5 |
| medium 4 | 0.208 | 126.9 | 4.21 | 15.0 | 45.0 |
| medium 5 | 0.251 | 163.7 | 4.57 | 22.2 | 74.1 |

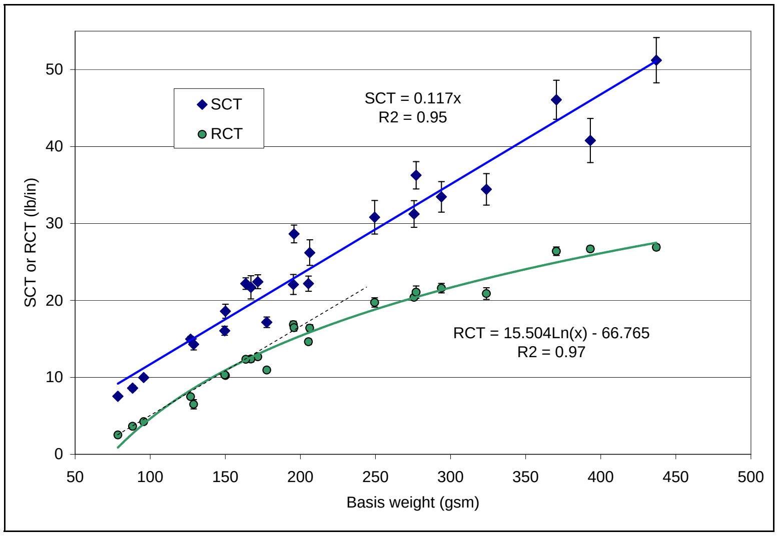

**Figure 4. Comparison of SCT and RCT with basis weight of samples.**

The ratio of buckling stress $\sigma_{CR}$ to the compression stress $SCT/t$ shown in Figure 5 indicates that the compression stress is equal to or exceeds the buckling stress at basis weights of 81 $g/m^2$ and less. For basis weights greater than 81 $g/m^2$, buckling exceeds compression stress so that the RCT specimen should be expected in principle, to fail by compression only. Note however, that using the linear elasticity theory result (1) to calculate the buckling stress is generally found by experiment to overestimate buckling stress values for thin-walled metal tubes by about a factor of 2 [8]. Nonetheless, even by the crude estimate represented in Figure 5, a mix of bending and compression failure can be expected to occur in the RCT of most samples, much as it does for the case of combined panel buckling and compression failure in BCT.

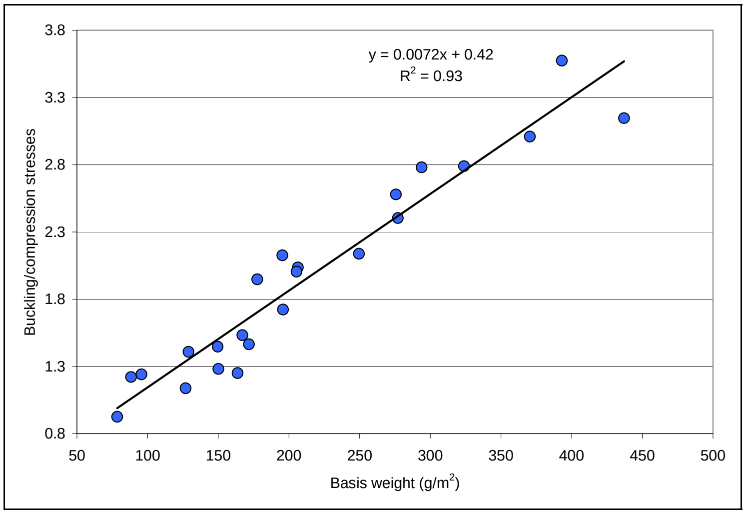

**Figure 5. Ratio of the buckling to the compression stress versus basis weight for the samples listed in Table 1.**

Several linear regression models for *RCT* were attempted using basis weight, bending stiffness, *RCT* and *TSI_CD*. The best agreement of models with actual *RCT* values comes using equation 5, which solving for the fitting constants *C* and *b* using the data for the sample set becomes:

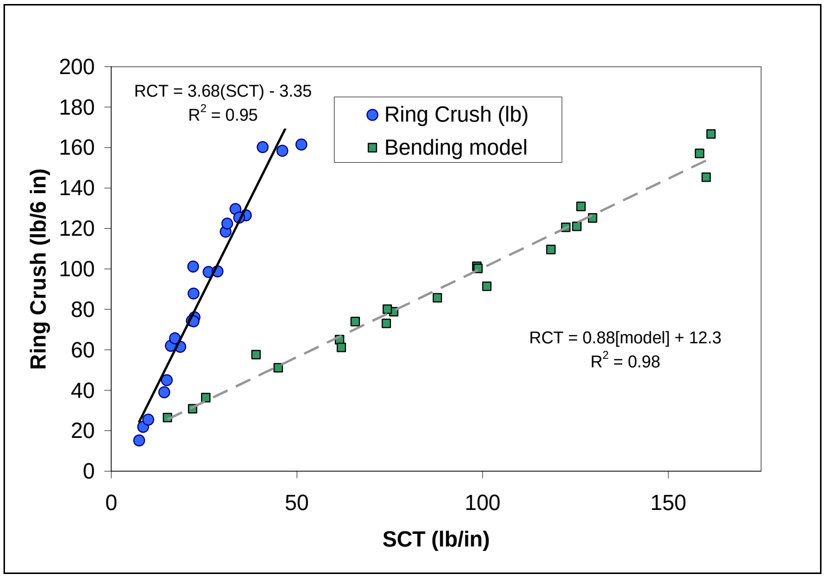

**Figure 6. Plots of RCT versus SCT and RCT versus the RCT bending model with fitted constants for Table 1 sample set.**

$$RCT = 0.35(SCT)^{.36}(TSI\_CD \times \beta)^{0.64} \qquad (7)$$

The correlation of the model (7) with actual RCT values is 0.98 which is an improvement from 0.95 the $r^2$ for RCT with SCT alone. Moreover, the mean square error between the model and actual RCT values is 1.6 lbs whereas the error for the linear fit of RCT with SCT is 2.3 lbs.

## CONCLUSION

RCT is a combination of compression and bending failure which can be modeled in a form similar to the familiar BCT McKee equation. Ultrasonic measurement of specific stiffness or velocity squared in the CD multiplied by the basis weight of the samples offers a convenient means of obtaining the tensile stiffness which is required in the bending/compression RCT model. This combination of results allow an accurate estimate of the RCT when SCT, basis weight and specific stiffness are available as in many automated testing stations currently installed throughout industry operations.

## ACKNOWLEDGMENT

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## REFERENCES

1) Whitsitt, W. J., Papermaking factors affecting box properties, *Tappi Journal,* Dec. (1988) 163-167.

2) McKee, R.C., Gander, J.W., Wachuta, J.R., Compression Strength Formula for Corrugated Boxes, *Paperboard Packaging* 1963*;* **48**(8): 149-159.

3) Cavlin, S., Fellers, C., A new method for measuring the edgewise compression properties of paper, *Svensk Papperstidning* 1975; **78** (9): 329-332.

4) Timoshenko, S.P., Gere, J.M., Theory of Elastic Stability, McGraw Hill Book Company, Singapore, 1961.

5) Baum, G.A., Brennan, D.C., Habeger, C.C., Orthotropic elastic constants of paper, *Tappi Journal* 1981; **64** (8): 97-101.

6) Waterhouse, J.F., Ultrasonic testing of paper and paperboard: principles and applications, *Tappi Journal* 1994; **77** (1): 120 – 126.

7) Carson, C.G., Popil, R.E., Examining interrelationships between caliper, bending and tensile stiffness of paper in testing validation" *Tappi Journal* 2008; **7**(12): 17-24.

8) Shallhorn, P., Ju, S., Gurnagul,N., A Model for the Ring Crush Test of Paperboard, *Journal of Pulp and Paper Science* 2005; **31**(3):143-146.